

TTIP impact in Ireland

Annex CGE model technical overview

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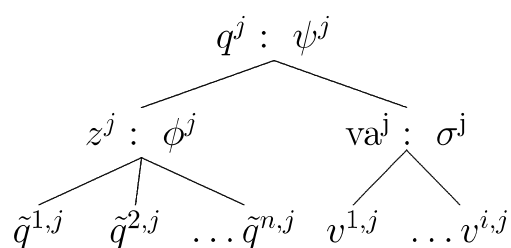
In the computational model, the "whole" economy, for the relevant aggregation of economic agents, is modelled simultaneously. This means that the entire economy is classified into production and consumption sectors. These sectors are then modelled collectively. Production sectors are explicitly linked together in value-added chains from primary goods, through higher stages of processing, to the final assembly of consumption goods for households and governments. These links span borders as well as industries. The link between sectors is both direct, such as the input of steel into the production of transport equipment, and also indirect, as with the link between chemicals and agriculture through the production of fertilizers and pesticides. Sectors are also linked through their competition for resources in primary factor markets (capital, labour, and land). The data structure of the model follows the GTAP database structure, and basic models of this class are implemented in either GEMPACK or GAMS (Hertel et al 1997, Rutherford and Paltsev 2000, Hertel 2013). We work here with a GEMPACK implementation.

Production

We start here with a representative production technology using a basic, constant returns to scale specification. Where we have scale economies, this serves as the cost structure for composite input bundles. Assume that output q^j in sector j can be produced with a combination of intermediate inputs z^j and value added services (capital, labour, land, etc.) va^j . This is formalized in equation 1. Assuming homothetic cost functions and separability, we can define the cost of a representative bundle of intermediate inputs z^j for the firm producing q^j and similarly the cost of a representative bundle va^j of value added services. These are shown in equations 2 and 3. They depend on the vector of composite goods prices \tilde{P} and primary factor prices ω . Unit costs for q then depend on the mix of technology and prices embodied in equations 1,2,3. We represent this in equation 4, which defines unit cost ζ^j . In the absence of taxes, in competitive sectors ζ^j represents both marginal cost and price. On the other hand, with imperfect competition on the output side (discussed explicitly later) ζ^j can be viewed as measuring the marginal cost side of the optimal markup equation, with markups driving a wedge between ζ^j and P^j .

To combine production technologies with data, we need to move from general to specific functional forms. We employ a nested CES function, with a CES representation of value added activities va^j , a CES representation of a composite intermediate z^j made up of intermediate inputs, and an upper CES nest that then combines these to yield the final good q^j . Our set-up is illustrated in Figure 1 below, on the assumption we have i primary factors v , as well as n production sectors that can be represented in terms of composite goods \tilde{q} as defined below.

Figure 1: representative nested production technology



These composites may (or may not, depending on the goods involved) be used as intermediate inputs. In Figure 1, we have also shown the CES substitution elasticity for intermediate inputs ϕ , the substitution elasticity for value added σ , and the substitution elasticity for our "upper nest" aggregation of value added and intermediates, ψ . In the absence of taxes, total value added Y will be the sum of primary factor income, as in equation 5.

Given our assumption of CES technologies, we can represent value added in sector j as a function of primary inputs and the elasticity of substitution in value added σ^j . This yields equation 6, and its associated CES price index shown in equation 7. Similarly, we can specify the CES price index for composite intermediates, as in equation 7. This gives us equation 8, where the coefficient ϕ^j is the elasticity of substitution between intermediate inputs. This is assumed to be Leontief (i.e. $\phi^j = 0$). Finally, following Figure 1, we will also specify an aggregation function for value added and intermediate inputs, in terms of its CES price index. This is shown as

equation 9. From the first order conditions for minimizing the cost of production, we can map the allocation of primary factors to the level of value added across sectors. This is formalized in equation 10. We can also specify the total demand for composite intermediate goods across sectors $\tilde{q}^{int,j}$ as a function of the producer price of composite input price P_{z^j} in each sector, the scale of intermediate demand across sectors z^j , and prices of composite goods \tilde{P}_r . This is shown in equation 11. With the upper nest CES for goods we can also map value added va^j and intermediate demand z^j in terms of equations 7 and 8, output q^j and the elasticity of substitution ψ^j between inputs and value added. This yields equations 12 and 13, where the terms γ are the CES weights (similar to those in equation 6) while ψ^j is the upper nest elasticity of substitution in the production function.

We also model some sectors as being characterized by large group monopolistic competition. In reduced form, this can be represented by an industry level scale economy that reflects variety effects. We define the price of output at industry level as in equation 14. In this case, ζ^j is defined by equation 9 and represents the price of a bundle of inputs, and equation 14 follows directly from average cost pricing, homothetic cost functions, and Dixit-Stiglitz type monopolistic competition. (See Francois and Roland-Holst 1997, Francois 1998, and Francois, van Meijl, and van Tongeren 2005 for explicit derivations.)

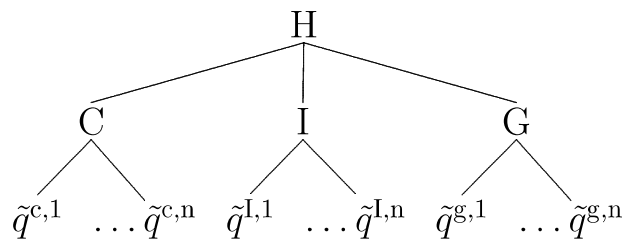
Together, equations 1 through 14 map out the production side of the economy. For an open economy, given resources, technology (represented by technical coefficients in the CES functional forms), and prices for foreign and domestic goods and services, we can determine factor incomes, national income, and the structure of production. We close this system by discussion of the demand side of the economy, and basic open economy aspects, in the next sections.

Final Demand

In the system we have spelled out so far, we have mapped the basic, national structure of production. We close the system with a demand specification for a representative household. This involves allocation of regional income by the household to composite

consumption H , which is separated over private consumption C , public consumption G , and investment I . Each of these components of H involves consumption of composite goods and services \tilde{q} indexed by sector j . This is illustrated in Figure 2 below. Where we assume fixed expenditure shares (i.e. with H taking a Cobb-Douglas functional form), then we also have a fixed savings rate. Otherwise, given the equilibrium allocation of household income to consumption and investment, we will denote these expenditure shares by θ . We maintain a fixed-share allocation between public and private consumption.

Figure 2: representative household demand



We assume a well-defined CES utility function for personal consumption defined over goods \tilde{q} . From the first order conditions for utility maximization, we can then derive the price of utility from private consumption P_U as a function of prices \tilde{P} , as in equation 15. The corresponding expenditure function is then $U = U^c P_U$ where U^c is the level of utility from private consumption. Taking national income as our budget constraint, then combining equation 5 with the expenditure function yields equation 16. From 16, we can define U^c from the expenditure function and income, as in equation 17. Consumption quantities, in terms of composite goods, can be recovered from equation 17, as shown in equation 18. Like private consumption, the public sector is also modelled with a CES demand function over public sector consumption. This implies equations 19-22. For investment demand, in the short run, we assume a fixed savings rate. In the long-run, the model can alternatively incorporate a fixed savings rate, or a rate that adjusts to meet steady state conditions in a basic Ramsey structure with constant relative risk aversion (CRRA) preferences. We employ the CRRA version here. (Francois, McDonald and Nordstrom 1996). With fixed savings,

and assuming a Leontief composite of investment goods that make up the regional investment good, investment demand is defined by equation 23. With CRRA preferences, steady-state conditions implies equation 24 as well, related to the price of capital ω_k . Where 24 holds, the additional equation allows us to make the savings rate coefficient θ^l endogenous. In equation 24 ρ is the rate of time discount and δ is the rate of depreciation. With a short-run or static closure, investment demand means we apply equation 23. With a long-run closure, we also apply equation 25. When we impose CRRA preferences in the long-run, we then employ all three equations on the model 23-25, and savings rates are endogenous. With a fixed savings rate, we drop equation 24 and make θ^l exogenous.

Cross-border linkages and taxes

Finally, individual countries, as described by equations 1-25 above, are linked through cross border trade and investment flows. With either monopolistic competition or Armington preferences, we can define a CES composite good \tilde{q} in terms of foreign and domestic goods. The price index for this composite good is defined by equation 26. Given equation 26 and the envelope theorem, we can define domestic absorption D as in equation 27, where h indexes home prices and quantities. The difference between production q_j and domestic absorption D_j in equilibrium will be imports (where a negative value denotes exports), as in equation 28. Across all countries indexed by r , we also have a global balanced trade requirement, shown in equation 29. Similarly, balancing the global capital account also requires equations 30 and 31 (where we now index source r and home destination h).

Trading costs are modelled as in ECORYS (2009), Copenhagen Economics (2009), and CPER (2013) and benchmark values for NTBs come from ECORYS and CEPR. Information on the extent to which policies affect prices and costs is important for accurate modelling of policy reforms, including whether policies create "rents" as opposed to being resource-using (generating "waste"), and the identity (ownership) of the entities and groups to whom any rents accrue. This is a well-known issue that can have a major bearing on the magnitude of the welfare impacts of policies and policy reforms. For example, if a policy generates rents for domestic groups and liberalization results in a share of these rents accruing to foreign entrants, the result

may be lower national welfare. Recent work supported by the EC (ECORYS 2009, Copenhagen Economics 2009, CEPR 2013) has been focused explicitly on this distinction, and the results of this analysis feed into the estimates reported in this study. In the estimates below, we distinguish between cost and rent generation under NTMs. Rents are modelled, in effect, like export and import taxes. For cost-raising barriers, we follow the now standard approach to modelling iceberg or dead-weight trade costs in the GTAP framework, originally developed by Francois (1999, 2001) with support from the EC to study the Millennium Round (now known as the Doha Round). This approach has grown from an extension in early applications to a now standard feature of the GTAP model, following Hertel, Walmsley and Itakura (2001). It has featured in the joint EC-Canadian government study on a EU-Canada FTA, as well as the 2009 ECORYS study on EU-US non-tariff barriers. In formal terms, changes in the value of this technical coefficient capture the impact of non-tariff measures on the price of imports from a particular exporter due to destination-specific reduced costs for production and delivery., while cost raising barriers are modelled

The basic system outlined above provides the core production and demand structure of each region, as well as the basic requirements for bilateral import demand, global market clearing for traded goods and services, and global capital account balancing. Within this basic structure, we also introduce taxes, transport services, iceberg (deadweight) non-tariff barriers, and rent-generating non-tariff barriers. These drive a wedge between the ex-factory price originating in country r and the landed prices in country h inclusive of duties and transport costs. Taxes and rent-generating trade costs mean that Y is also inclusive of tax revenues and rents. In the short-run we fix B , while in the long-run this is endogenous (such that the distribution of relative global returns is maintained). All of this adds additional complexity to the system outlined above, but the core structure remains the same.

Key elasticities and sector specifications are summarized below.

Sector level elasticities

MC sectors		trade price elasticity	value added substitution elasticity
	1 Agforfish	4.7664	0.3
	2 Beef	7.03	0.67
	3 Dairy	7.3	0.69
	4 ProcFood	2.46	1.12
	5 Energy	10.12	0.63
MC	6 PharmaChem	5.09	1.26
MC	7 Elemach	9.65	1.26
MC	8 Omach	9.71	1.26
	9 Vehicles	10	1.26
	10 OTspeqp	7.14	1.26
	11 Metals	13.91	1.26
	12 WoodPap	7.99	1.26
MC	13 OtherMfg	6.5558	1.26
	14 Airtsp	3.8	1.68
	15 Watertsp	3.8	1.68
	16 OtherTsp	3.8	1.68
	17 Finance	2.04	1.26
	18 Insurance	3.18	1.26
MC	19 BusICT	3.18	1.26
	20 Comm	3.18	1.26
	21 Const	4.21	1.4
	22 Personal	8.71	1.26
	23 OthServices	3.92	1.39

Source: these elasticities are based on ECORYS(2009).

MC sectors are the same as those for the ECORYS(2009) study.

$$(1) \quad q^j = f^j(z^j, \text{va}^j)$$

$$(2) \quad P_z = g(\tilde{P})$$

$$(3) \quad P_{\text{va}} = h(\omega)$$

$$(4) \quad \zeta_j = c(P_z, P_{\text{va}})$$

$$(5) \quad Y = \sum_i \omega_i v_i$$

$$(6) \quad \text{va}^j = \left[\sum_i \alpha_{ij} v_{ij}^{\frac{\sigma^j-1}{\sigma^j}} \right]^{\frac{1}{\sigma^j-1}}$$

$$(7) \quad P_{v^j} = \left[\sum_i \alpha_{ij}^{\sigma^j} \omega_i^{1-\sigma^j} \right]^{\frac{1}{1-\sigma^j}}$$

$$(8) \quad P_{z^j} = \left[\sum_i \beta_{ij}^{\phi^j} \tilde{P}_i^{1-\phi^j} \right]^{\frac{1}{1-\phi^j}}$$

$$(9) \quad P_j = \left(\gamma_{vj}^{\psi^j} P_{\text{va}^j}^{1-\psi^j} + \gamma_{zj}^{\psi^j} P_{z^j}^{1-\psi^j} \right)^{\frac{1}{1-\psi^j}}$$

$$(10) \quad v_i \geq \sum_j \text{va}^j \left(\frac{\alpha_{vj}}{\omega_i} \right)^{\sigma^j} P_{\text{va}^j}$$

$$(11) \quad \tilde{q}^{\text{int},i} = \sum_j z^j \left(\frac{\beta_{ij}}{\tilde{P}_i} \right)^{\phi^j} P_{z^j}$$

$$(12) \quad \text{va}^j = q^j \left(\frac{\gamma_{vi}}{P_{v^j}} \right)^{\psi^j} P_j$$

$$(13) \quad \bar{z}^j = q^j \left(\frac{\gamma_{zi}}{P_{z^j}} \right)^{\psi^j} P_j$$

$$(14) \quad P_j = q_j^{\psi} \left(\gamma_{vj}^{\psi^j} P_{\text{va}^j}^{1-\psi^j} + \gamma_{zj}^{\psi^j} P_{z^j}^{1-\psi^j} \right)^{\frac{1}{1-\psi^j}}$$

where $1 > \psi > 0$

$$(15) \quad P_{U^c} = \left(\sum_{i=1}^n \alpha_{c,i}^{\eta^c} \tilde{P}_i^{1-\eta^c} \right)^{\frac{1}{1-\eta^c}}$$

where $0 < \frac{\eta^c-1}{\eta^c} < 1$

$$(16) \quad U^c \left(\sum_{i=1}^n \alpha_{c,i}^{\eta^c} \tilde{P}_i^{1-\eta^c} \right)^{\frac{1}{1-\eta^c}} = Y \theta^c$$

$$(17) \quad U^c = \left(\sum_{i=1}^n \alpha_{c,i}^{\eta^c} \tilde{P}_i^{1-\eta^c} \right)^{\frac{1}{\eta^c-1}} Y \theta^c$$

$$(18) \quad \tilde{q}^{c,i} = U^c P_{U^c}^{\eta^c} \alpha_{c,i}^{\eta^c} \tilde{P}_i^{-\eta^c}$$

$$(19) \quad P_{U^g} = \left(\sum_{i=1}^n \alpha_{g,i}^{\eta^g} \tilde{P}_i^{1-\eta^g} \right)^{\frac{1}{1-\eta^g}}$$

where $0 < \frac{\eta^g-1}{\eta^g} < 1$

$$(20) \quad U^g \left(\sum_{i=1}^n \alpha_{g,i}^{\eta^g} \tilde{P}_i^{1-\eta^g} \right)^{\frac{1}{1-\eta^g}} = Y \theta^g$$

$$(21) \quad U^g = \left(\sum_{i=1}^n \alpha_{g,i}^{\eta^g} \tilde{P}_i^{1-\eta^g} \right)^{\frac{1}{\eta^g-1}} Y \theta^g$$

$$(22) \quad \tilde{q}^{g,i} = U^c P_{U^g}^{\eta^g} \alpha_{g,i}^{\eta^g} \tilde{P}_i^{-\eta^g}$$

$$(23) \quad \left(\sum_{j=1}^n \alpha_{I,j} \tilde{P}_j \right) = Y \theta^I$$

$$(24) \quad \omega_k = P^c (\rho + \delta)$$

$$(25) \quad dK/K = dI/I$$

$$(26) \quad \tilde{P}_j = \left(\sum_{r=1}^R b_{r,j}^{s^j} P_{r,j}^{1-s^j} \right)^{\frac{1}{1-s^j}}$$

where $0 < \frac{s^j-1}{s^j} < 1$

$$(27) \quad D_j = (\tilde{q}^{c,j} + \tilde{q}^{l,j} + \tilde{q}^{g,j} + \tilde{q}^{\text{int},i}) \tilde{P}_j^s b_{h,j}^s P_{h,j}^{-s}$$

$$(28) \quad M_j = D_j - q_j$$

$$(29) \quad \left(\sum_{r=1}^{rr} M_{r,j} \right) = 0$$

$$(30) \quad \left(\sum_j \sum_{r \neq h} P_{r,j} M_{r,h,j} \right) = B_h$$

$$(31) \quad \left(\sum_r B_r \right) = 0$$

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